Three-dimensional forward modelling and inversion of Z-TEM data

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SUMMARY

Z-TEM data are airborne EM data which record the vertical magnetic field that results from natural sources. Processing the data yields transfer functions which relate the vertical magnetic field to horizontal fields at a reference station. The transfer functions depend upon frequency and provide information about the 3D conductivity structure of the Earth. In this paper we show how these transfer functions can be modelled. This is the crucial step towards any interpretation or inversion. The inversion of Z-TEM data is computationally similar to inversion of MT data. This ability will enhance exploration efforts to find large scale structures buried at depth.

INTRODUCTION

Magnetotellurics (MT) is a passive electromagnetic exploration technique that is sensitive to deep conductivity structures. Because of the ability to image a large range of depths, MT plays a significant role in both crustal studies as well as mining and hydrocarbon exploration. As the number of remaining near surface exploration targets decreases, MT techniques play an increasing role in exploration. This trend, combined with increased computational abilities, has contributed to significant work in three-dimensional MT modelling. One practical limitation of the traditional technique is the time consuming and costly nature of the survey. This is because multiple expensive stations must be installed to measure all of the field components at the surface of the earth.

A technique that shows promise for discovering larger scale structures is the Z-TEM system. Currently Z-TEM data is being collected by both industry and government. This technique is very similar to the MT technique however in a Z-TEM survey only the magnetic fields are recorded. The vertical fields are recorded in an airborne method along the survey lines while the horizontal fields are recorded at one grounded base station. The data to be interpreted are the transfer functions that relate the vertical component of the magnetic field to the horizontal field at the base station. In order to interpret data we need to forward model the responses and ultimately invert them. In this paper we present a method to compute Z-TEM transfer functions as well as a method to invert them.

FORWARD MODELLING

Our forward modelling procedure is based upon the MT works of Farquharson et al. (2002), where the solution of Maxwell's equations is that of Haber et al. (2000a). The essential details are summarized below.

The electric field is decomposed into vector and scalar poten-

tials

$$\mathbf{E} = \mathbf{A} + \nabla\phi, \tag{1}$$

and the Coloub gauge condition

$$\nabla \cdot \mathbf{A} = 0, \tag{2}$$

is imposed for uniqueness. Maxwell's equations combined with our decomposition gives

$$\nabla^2\mathbf{A} + i\omega\mu_0\sigma(\mathbf{A} + \nabla\phi) = 0, \tag{3}$$

where ω is the angular frequency, μ_0 is the magnetic permeability of free space, $\sigma = \sigma(r)$ is the electrical conductivity of our Earth model and $i = \sqrt{-1}$. We have also used the quasi-static assumption and assumed a $e^{-i\omega t}$ time-dependence. Current densities are given according to

$$\mathbf{J} = \sigma(\mathbf{A} + \nabla\phi), \tag{4}$$

and

$$\nabla \cdot J = 0. \tag{5}$$

For our modelling we discretize the earth into rectangular cells and, after applying a finite volume technique to equations 3-5 Haber et al. (2000a), we obtain a system of equations to be solved.

$$\begin{pmatrix} L + i\omega\mu_0 S & i\omega\mu_0 SG \\ DS & DSG \end{pmatrix} \begin{pmatrix} A \\ \phi \end{pmatrix} = 0. \tag{6}$$

A now represents the vector of components of the vector potential on the mesh, and ϕ represents the vector of scalar potentials. Furthermore, L represents the discretization of the Laplacian operator, S represents the averaged cell conductivities, and G and D are the discretizations of the gradient and divergence operators. For each source polarization (one for an x-directed H-field at the top of the mesh, and the other for a y-directed H-field) the resulting E and H fields are computed.

MAGNETIC TRANSFER FUNCTIONS

Transfer functions relate the vertical magnetic fields computed above the earth to the horizontal magnetic field at some fixed reference station. This relation is given by

$$H_z(r) = T_{zx}(r, r_0)H_x(r_0) + T_{zy}(r, r_0)H_y(r_0), \tag{7}$$

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where r is the location for the vertical field, and r_0 is the location of the ground based reference station.
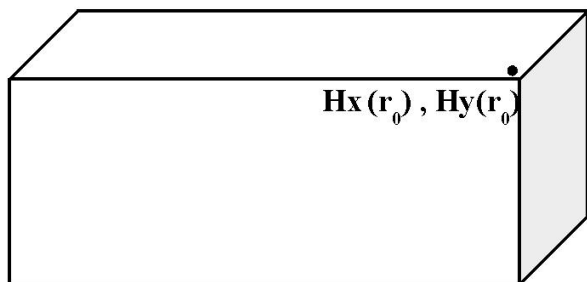
- **Hz (r)**

Figure 1: Locations of the data required for the computation of the magnetic transfer functions. The vertical component of the fields are collected at a height above the surface of the earth. The position **r**, is the location of a single vertical field measurement. The horizontal fields are obtained at one fixed reference station located at r_0, on the earth. This reference station should be located just outside of the region of interest.

Our source functions for the natural fields are random and as with MT, we need two polarizations. The transfer functions for each polarization are given by

$$H_z^{(1)} = T_{zx}H_x^{(1)} + T_{zy}H_y^{(1)}, \qquad (8)$$

$$H_z^{(2)} = T_{zx}H_x^{(2)} + T_{zy}H_y^{(2)}, \qquad (9)$$

or

$$\begin{pmatrix} H_z^{(1)}(r) \\ H_z^{(2)}(r) \end{pmatrix} = \begin{pmatrix} H_x^{(1)}(r_0) & H_y^{(1)}(r_0) \\ H_x^{(2)}(r_0) & H_y^{(2)}(r_0) \end{pmatrix} \begin{pmatrix} T_{zx} \\ T_{zy} \end{pmatrix}, \qquad (10)$$

where the superscripts (1) and (2) refer to the source field polarization in the x and y directions respectively. These two relations can be combined to yield the equations that must be solved for the transfer functions:

$$\begin{pmatrix} T_{zx} \\ T_{zy} \end{pmatrix} = \begin{pmatrix} H_x^{(1)}(r_0) & H_y^{(1)}(r_0) \\ H_x^{(2)}(r_0) & H_y^{(2)}(r_0) \end{pmatrix}^{-1} \begin{pmatrix} H_z^{(1)}(r) \\ H_z^{(2)}(r) \end{pmatrix}. \qquad (11)$$

EXAMPLE

Transfer function computation

We illustrate the computation of magnetic transfer functions using a synthetic example that was chosen to emulate a porphyry deposit. The model has a central resistive core, an outer region of high conductivity, and is buried in a background host. The conductivity structure is shown in figure 2.
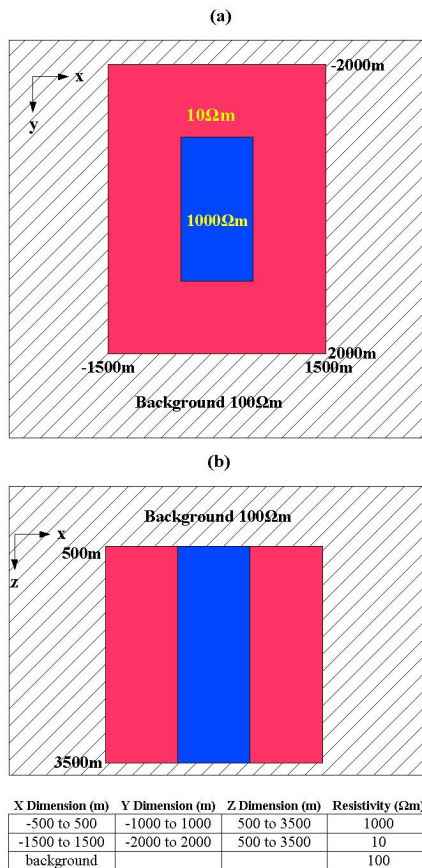

Figure 2: Conductivity model: a resistor surrounded by a conductor, all buried in a 100Ωm background halfspace. Panel (a) shows a plan view and panel (b) shows a vertical section through the blocks. The bottom table summarizes the block dimensions and resistivites.

X Dimension (m)	Y Dimension (m)	Z Dimension (m)	Resistivity (Ωm)
-500 to 500	-1000 to 1000	500 to 3500	1000
-1500 to 1500	-2000 to 2000	500 to 3500	10
background			100

The Earth was discretized into a mesh containing 68 x 68 x 70 cells in the x, y, and z-directions respectively. A single 20Hz signal was used. The vertical magnetic fields were computed at a fixed height of 20m over the surface to emulate the data collected by an airborne survey. The data were computed at 100m intervals over a 2.3 km by 2.3 km region. The horizontal field components were computed at a ground based reference station at a location r_0 of x = - 2600m, y = - 2600m.

Data visualization

The transfer functions are complex. They have real and imaginary parts that change with frequency. The transfer functions for the synthetic example are displayed in figure 3. In these simple circumstances the data display structure that is associated with the outer boundaries of the blocks. There is no visible depth information with the plot. We can get some further
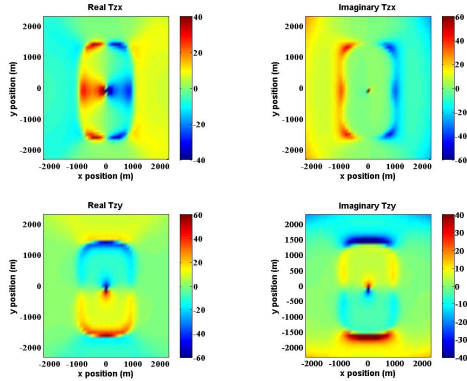
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Figure 3: Real and imaginary components of the magnetic transfer functions Tzx and Tzy.
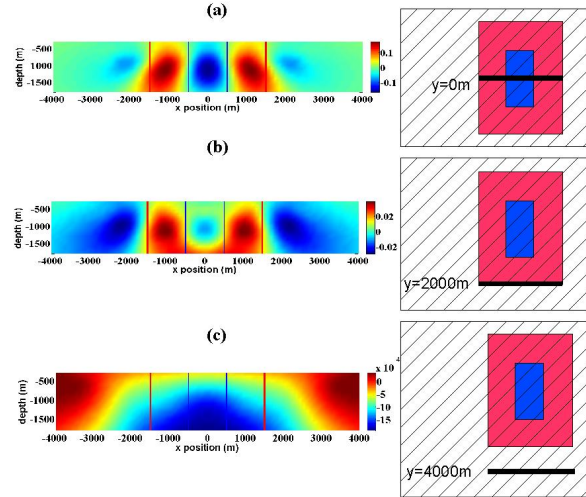

Figure 4: Pseudosection from linear filtering the vertical magnetic fields computed from a x-directed H source field. The synthetic earth model was the resistor embedded in a conductor seen in figure 2. The pseudosection gives a relative current density along three cross sections in the model. The block boundaries have been superimposed on the plots to help aid the interpretations. Panel (a) is a measure of relative current density vs depth for the profile y = 0m. This profile intersects the middle of the conductor and resistor as can be seen in the miniature plan view model to the right of the pseudosection. Panel (b) is a current cross section along the profile y = 2000m which is along the edge of the conducting block. Panel (c) is along the profile y = 4000m, well into the background halfspace.

idea about depth and boundaries through filtering techniques. One such filtering scheme is that of Karous and Hjelt (1983) in which linear filtering techniques of vertical magnetic fields can be used to generate current pseudosections. We show that these pseudosections can often aid to provide satisfactory interpretations of simple models.

The current pseudosections obtained from linear filtering the vertical magnetic fields from the previous synthetic example are shown in Figure 4. The fields were computed for a x-directed source H field. The plots were generated using the Matlab code of Sundararajana et al. (2006). As one would expect along the profile y = 0m, the current flow is opposite in the conductor and resistor regions. As we move to y = 2000m, the central resistor response has diminished since we are now at the edge of the conductor. Finally as we move well into the halfspace we have very little current density throughout the entire cross-section. These pseudosections help us define the block boundaries at depth; however, this method is only an approximation. Nonetheless, these pseudosections when combined with the information gained from plotting the transfer functions, can provide some interpretation for simple model geometries.

INVERSION

Our inversion algorithm is that of Farquharson et al. (2002) and is implemented by the minimization of the data-misfit and model complexity. The solution is obtained by an iterative Gauss-Newton procedure based on Haber et al. (2000b).

By minimizing the objective function

$$\Phi = \phi_d + \beta \phi_m, \qquad (12)$$

we obtain our solution to the inverse problem. ϕ_d is the data-misfit, ϕ_m is amount of structure in the model, and β is the trade-off or regularization parameter. We use the sum of squares as the measure of misfit

$$\phi_d = ||\mathbf{W}_d(\mathbf{d}^{obs} - \mathbf{d}^{prd})||_2^2, \qquad (13)$$

where $\mathbf{d}^{obs}$ is the observation vector, $\mathbf{W}_d$ is a diagonal matrix whose elements are the reciprocals of the measurement uncertainties. The measure of model structure is

$$\phi_m = \sum_{k=1}^{4} \alpha_k ||\mathbf{W}_k(\mathbf{m} - \mathbf{m}^{ref})||_2^2, \qquad (14)$$

where $\mathbf{W}_1$ is a diagonal matrix and $\mathbf{W}_2, \mathbf{W}_3$ and $\mathbf{W}_4$ are first order finite-difference matrices in the x, y and z directions, and $\mathbf{m}^{ref}$ is a reference model. For the minimization of the objective function at the (n+1)th iteration, the Gauss-Newton method requires the solution of

$$(\mathbf{J}^T \mathbf{W}_d^T \mathbf{W}_d \mathbf{J} + \beta \mathbf{W}^T W) \delta \mathbf{m} =$$
$$-\mathbf{J}^T \mathbf{W}_d^T \mathbf{W}_d (\mathbf{d}^{obs} - \mathbf{d}^n) - \beta \mathbf{W}^T \mathbf{W}(\mathbf{m}^n - \mathbf{m}^{ref}), (15)$$

where $\mathbf{m}^n$ is the vector of model parameters from the previous iteration, $\mathbf{J} = \mathbf{J}(\mathbf{m}^n)$ is the Jacobian matrix of sensitivities, W

is such that $\mathbf{W}^T\mathbf{W} = \sum \alpha_k \mathbf{W}_k^T \mathbf{W}_k$, and $\delta\mathbf{m}$ is the perturbation to be added to $\mathbf{m}^n$ to give the new model.

At present our inversion algorithm is only implemented to invert magnetotelluric impedances; however, with modifications we will be able to invert magnetic transfer functions.

CONCLUSIONS

Z-TEM airborne data is currently being collected and provides new potential for discovering larger scale structures. After processing, the data are transfer functions that are a function of frequency. We have shown how to simulate these forward responses on a simple synthetic example. The next step is to invert them to recover a 3D conductivity model. The methodology has been established, and we are pursuing this goal.

EDITED REFERENCES
Note: This reference list is a copy-edited version of the reference list submitted by the author. Reference lists for the 2008 SEG Technical Program Expanded Abstracts have been copy edited so that references provided with the online metadata for each paper will achieve a high degree of linking to cited sources that appear on the Web.

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