Three-dimensional inversion of MT and ZTEM data

*Elliot Holtham*and Douglas W. Oldenburg, UBC-Geophysical Inversion Facility, University of British Columbia*

SUMMARY

ZTEM is an airborne electromagnetic survey in which the vertical magnetic field from natural sources is recorded. The data are transfer functions that relate the local vertical field to orthogonal horizontal fields measured at a reference station on the ground. The transfer functions depend on frequency and provide information about the 3D conductivity structure of the Earth. Since a 1D conductivity structure produces no vertical magnetic fields, the ZTEM technique is not very sensitive to the background conductivity. In order to increase sensitivity to the background conductivity, and greatly improve the depth of investigation, MT and ZTEM data can both be collected. The combination of sparse MT data, with the economical and rapid spatial acquisition of airborne ZTEM data, creates a cost effective exploration technique that can map large-scale structures at depths that are difficult to image with other techniques. We develop a Gauss-Newton algorithm to jointly invert ZTEM and MT data. The algorithm is applied to a synthetic model and to a field example from the Reese River geothermal property in Nevada.

INTRODUCTION

Natural source electromagnetics have an important role in understanding the electrical conductivity of upper regions of the Earth. Their primary advantage, compared to controlled source methods, is the depth of penetration that is a consequence of the plane wave excitation. The magnetotelluric (MT) method uses ratios of electric and magnetic fields as data and it has played a significant role in crustal studies as well as in mining and hydrocarbon exploration. A practical limitation of the MT technique however, is that surveys are costly and time consuming because many expensive stations must be installed to measure all of the field components at the surface of the earth. It would be preferable to collect MT data in an aircraft but this goal has not yet been achieved because of the difficulty in measuring the electric fields. In an effort to continue to use the penetration advantage of natural sources, it has long been recognized that tipper data, the ratio of the local vertical magnetic field to the horizontal magnetic field, provide information about 3D electrical conductivity structure. It was this understanding that prompted the development of AFMAG (Audio Frequency Magnetics) Ward (1959). However, because the direction and strength of the inducing field varies with time, AFMAG results were not always repeatable. Limitations of the AFMAG technique were outlined in Ward et al. (1966).

Many of the AFMAG problems can be removed by using improved signal processing and instrumentation. This has resulted in the Z-Axis Tipper Electromagnetic Technique (ZTEM) Lo and Zang (2008). In ZTEM, the vertical component of the magnetic field is recorded above the entire survey area, while the horizontal fields are recorded at a ground-based reference station. MT processing techniques yield frequency domain transfer functions typically between 30-720 Hz that relate the vertical fields over the survey area to the horizontal fields at the reference station. By taking ratios of the two fields (similar to taking ratios of the E and H fields in MT), the effect of the unknown source function is removed. Since new instrumentation exists to measure the vertical magnetic fields by helicopter, data over large survey areas can quickly be collected. The result is a cost effective procedure for collecting natural source EM data that provide information about the 3D conductivity structure of the earth. The technique is particularly good for finding large scale targets at moderate depths. It is well known however that 1D layered structures produce zero vertical magnetic fields and thus ZTEM data cannot recover such background conductivities. If the background structure is 3D there will be an effect on the data and correspondingly ZTEM data will contain information about the conductivity. In general however, ZTEM data may have reduced sensitivity about the background conductivity and it is desirable to obtain robust conductivity information from other sources. This information could come from a priori geologic and petrophysical information or from additional geophysical data such as MT.

To counter the costly nature of large MT surveys and the insensitivity of the ZTEM technique to the background conductivity, an effective exploration survey can collect both MT and ZTEM data. A sparse MT survey grid can gather information about the background conductivity and deep structures while keeping the survey costs affordable. Higher spacial resolution at moderate depths can be obtained by flying multiple lines of ZTEM data. In this paper we show how MT and ZTEM data can be jointly inverted in three dimensions to provide cost effective exploration solutions.

ZTEM DATA

The ZTEM data relate the vertical magnetic fields computed above the Earth to the horizontal magnetic field at some fixed reference station. This relation is given by

$$H_z(r) = T_{zx}(r,r_0)H_x(r_0) + T_{zy}(r,r_0)H_y(r_0), \tag{1}$$

where r is the location for the vertical field, and r_0 is the location of the ground based reference station. Our source functions for the natural fields are random and, as with MT, we need two polarizations. The transfer functions for each polarization are given by

$$\begin{pmatrix} H_z^{(1)}(r) \\ H_z^{(2)}(r) \end{pmatrix} = \begin{pmatrix} H_x^{(1)}(r_0) & H_y^{(1)}(r_0) \\ H_x^{(2)}(r_0) & H_y^{(2)}(r_0) \end{pmatrix} \begin{pmatrix} T_{zx} \\ T_{zy} \end{pmatrix}, \tag{2}$$

where the superscripts (1) and (2) refer to the source field polarization in the x and y directions respectively.

INVERSION

Our MT inversion algorithm is that of Farquharson et al. (2002) and our ZTEM inversion algorithm is that of Holtham and Oldenburg (2008) and Holtham and Oldenburg (2010). Our algorithm to invert MT and ZTEM data simultaneous was obtained by modifying these codes. The solution is obtained by an iterative Gauss-Newton procedure based on Haber et al. (2000).

By minimizing the objective function

$$\Phi = \phi_d + \beta \phi_m, \tag{3}$$

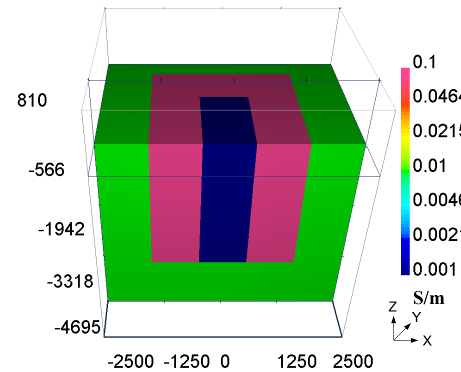
we obtain our solution to the inverse problem. ϕ_d is the datamisfit, ϕ_m is amount of structure in the model, and β is the trade-off or regularization parameter. Although the forward problems for ZTEM and MT are extremely similar, when considering the data there are many differences that can have important implications for the inversion process. Firstly the surveys are often performed at different times with significant differences in the source strengths and polarizations. Furthermore, because MT data are collected on the ground while ZTEM data are collected in the air, the number of data from each survey and the data errors can be drastically different. Our data misfit is computed via,

$$\phi_d = ||\mathbf{W}_{d1}(\mathbf{d}^{obs}_{ZTEM} - \mathbf{d}^{prd}_{ZTEM})||^2_2 + \gamma||\mathbf{W}_{d2}(\mathbf{d}^{obs}_{MT} - \mathbf{d}^{prd}_{MT})||^2_2, \tag{4}$$

where γ is a constant that must be determined. Although determining the optimal weights for different data types in an inversion is a difficult problem, we adopted the approach that MT and ZTEM should play approximately equal roles in the inversion. The constant was thus determined by inverting the MT and ZTEM individually before scaling each data set such that the final data misfits are equal.

SYNTHETIC EXAMPLE

We demonstrate the effectiveness of jointly working with MT and ZTEM data by examining the synthetic model in figure 1. The model has a central resistive core, an outer region of high conductivity, all intruded in a background host. The Earth was discretized into a mesh containing 56, 56, and 70 cells in the x, y, and z-directions respectively. MT data (0.04, 0.1, 0.4, 1, 3.5, 7, 30, 45, 90, 180, 360 Hz) and ZTEM data (30, 45, 90, 180, 360 Hz) were computed from the synthetic model using three different survey configurations over an approximate 10 x 10 km area. The first survey is a coarse MT survey with 72 stations in total with 1 km station spacing and 1 km line spacing. This significant station spacing will not be adequate to achieve the high resolution required for many detailed exploration projects. The second survey configuration is a finer MT survey with 1564 stations in total with 200 m station spacing and 300 m line spacing. Having over 1500 stations will be prohibitively expensive for most exploration programs considering a typical MT station can cost well in excess of $1000. The final ZTEM survey has 10 m data spacing and 300 m line spacing. Gaussian noise was added to all the data before being inverted. In addition, to demonstrate the advantages of working with sparse MT and ZTEM, the data from the coarse MT



X Dimension (m)	Y Dimension (m)	Z Dimension (m)	Resistivity (Ωm)
-500 to 500	-1000 to 1000	250 to 3500	1000
-1500 to 1500	-2000 to 2000	250 to 3500	10

Figure 1: Synthetic conductivity structure of a resistor surrounded by a conductor except on the top and bottom. Both blocks are embedded in a 100 Ωm background halfspace. Block dimensions are summarized in the accompanying table.

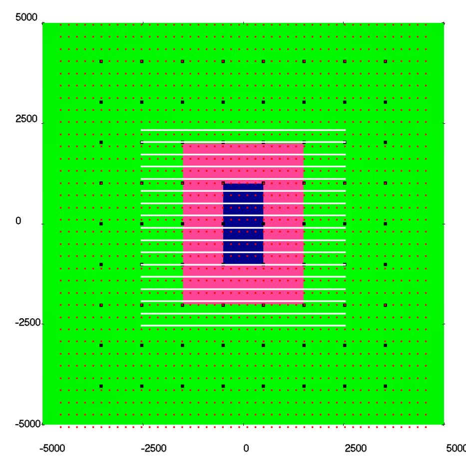


Figure 2: Survey grids for the synthetic inversions. The three grids are the coarse MT (black squares), fine MT (red squares), and ZTEM (white lines).

grid and the ZTEM grid were jointly inverted. For all the inversions, the predicted data fit the observed synthetic data very well. All of the inversions came close to achieving the target misfits. The final conductivity models can be seen in figure 3.

The coarse MT inversion result (figure 3b) recovers a crude estimate for the outline of the conductive block. Because of the limited data coverage from the 1 km x 1 km MT station spacing, the boundary of the blocks are not very well resolved. Nevertheless, the limited number of MT stations is enough to resolve the conductive block down to approximately 2500 m. By increasing the number of MT stations the boundaries of the blocks become better resolved (figure 3c). Had traditional MT tipper data been used, the additional information from the vertical fields may have improved the fine MT inversion result. The ZTEM inversion (figure 3d) shows good resolution down to almost 1000 m which is impressive for an airborne

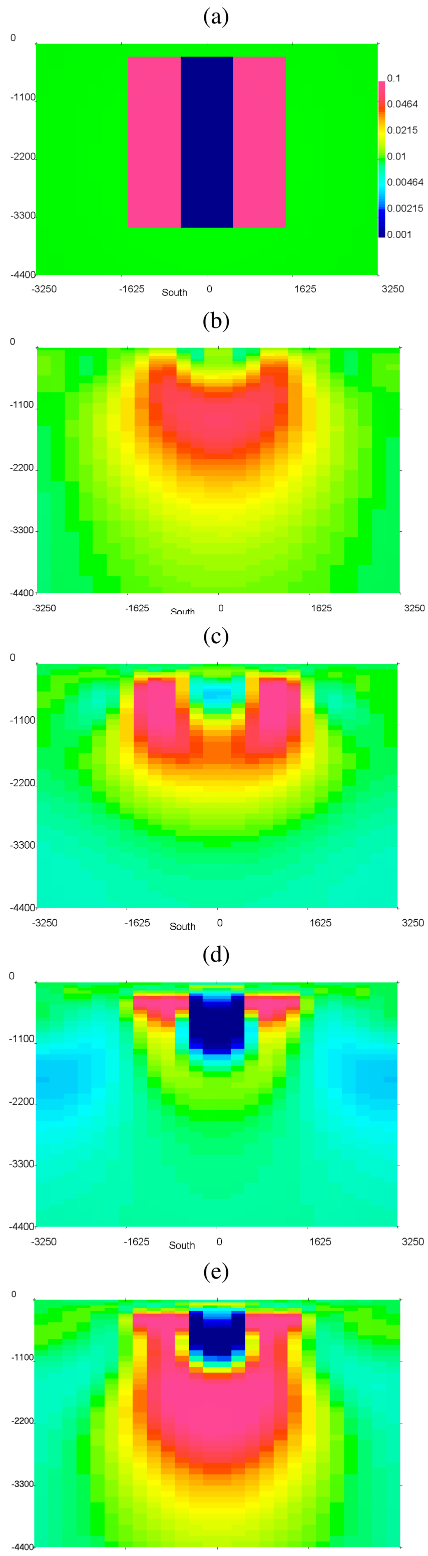


Figure 3: Inversion of the synthetic model in figure 1 all shown on the same color scale. a) True cross section of the model. Inversion results of the coarse MT grid (b), fine MT grid (c), ZTEM data (d), and joint coarse MT and ZTEM (e). Adding the ZTEM data to the coarse MT data greatly improved the inversion result.

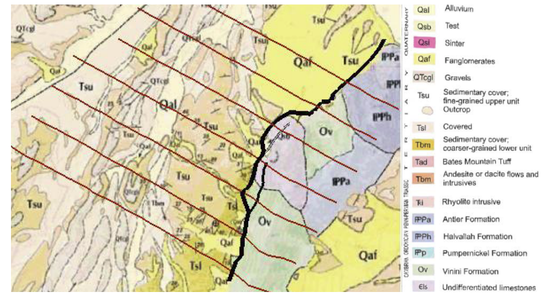


Figure 4: Reese River surface geology. The outline of the north south trending fault is shown in black. The MT lines are also shown.

technique. By jointly inverting the coarse MT grid and ZTEM grid (figure 3e), the inversion recovers both the deep structure from the low frequency MT data and fine structure from the ZTEM data. The 1 km x 1 km station spacing which was inadequate to recover the near-surface details is ideal to recover large scale features at the 2-3 km depth range. Interestingly, as one might expect, improving the model in the near-surface also improves the model at depth and resolves the model down to 3.5 km instead of the 2.5 km without the ZTEM data.

FIELD DATA

The joint inversion of MT and ZTEM data is an excellent technique to search for geothermal targets because it has the capability to penetrate the highly conductive clay cap often associated with geothermal reservoirs (Pellerin et al. (1996)). We demonstrate this by inverting overlapping MT and ZTEM data from the Reese River geothermal site in Nevada.

Reese River Geology

The Reese River property is divided by a north-south trending fault. At the near-surface, to the west of the fault is a sedimentary basin while to the east of the fault are more resistive rocks such as limestones. The surface geology and fault can be seen in figure 4.

Inversion Result

For the inversion the Earth was discretized on a mesh that contained 76, 78, and 87 cells in the x, y and z directions respectively. The cell dimensions in the central core region were 100 x 83.3 x 25 m. The MT data (0.048, 0.14, 0.44, 1.3, 3.7, 7 Hz) and the ZTEM data (30, 45, 90, 180, 360, 720 Hz) were inverted individually before being jointly inverted. The relative weights of the MT and ZTEM data in the inversion were scaled such that they had approximately equivalent contributions to the data misfit. The initial background conductivity was set to be 20 Ωm and the topography was obtained from the USGS national elevation database.

The inversion took 3 days running on 3 Intel Xeon 2.33Ghz dual quad core processors. The recovered conductivity model maps the fault very well as can be seen from the inversion re-

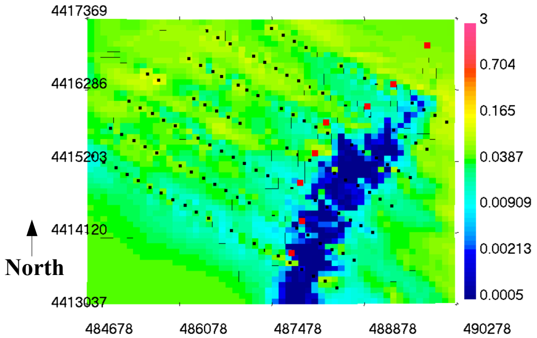


Figure 5: Conductivity model at a depth of 1800m which is the approximate average elevation of the fault boundary. The outline of the fault is shown by the red squares. The MT stations are shown in black. The conductivity model does a good job of showing the fault boundary.

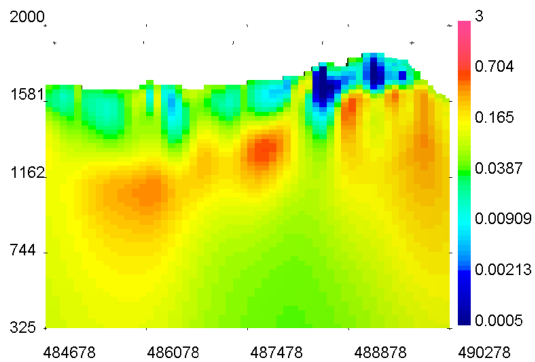


Figure 6: Cross section of the 3D conducitvity model along northing 4415620.

sult in figure 5. A slice of the model along northing 4415620 can be seen in figure 6. The inversion result seems to reproduce the limited geologic information available from the area.

CONCLUSIONS

MT, the traditional natural source electromagnetic method, has the greatest depth of investigation of all electromagnetic methods. Despite this, the costly nature of each MT station makes it impractical for large surveys that require high spacial resolution. Another natural source technique, ZTEM, is a promising method to explore for large scale targets at depth because of easy data acquisition and deep penetration of natural source fields. While the depths of investigation are limited somewhat by the lowest frequency signal that can be recorded in the air, the technique can still image to depths that are difficult to image with other controlled source airborne and ground based systems. One limitation of the technique is that there is little sensitivity to the background conductivity, especially as the Earth becomes a 1D medium.

The clear solution is to collect both MT and ZTEM data and exploit the strengths of each technique. The first step is to collect the minimal amount of MT data required to estimate the background conductivity and resolve the deep structure. High spacial resolution can then be achieved by flying multiple ZTEM lines over the sparse MT grid. The result is a cost effective solution that can map deep structures over large areas. We have developed a Gauss-Newton style inversion algorithm to jointly invert ZTEM and MT data. The algorithm shows promising results on both a synthetic test problem and a field example from the Reese River geothermal project.

ACKNOWLEDGEMENTS

The authors would like to thank Sierra Geothermal Power for their help on the Reese River Project

EDITED REFERENCES

Note: This reference list is a copy-edited version of the reference list submitted by the author. Reference lists for the 2010 SEG Technical Program Expanded Abstracts have been copy edited so that references provided with the online metadata for each paper will achieve a high degree of linking to cited sources that appear on the Web.

REFERENCES

Farquharson, C. G., D. W. Oldenburg, E. Haber, and R. Shekhtman, 2002, An algorithm for the three-dimensional inversion of magnetotelluric data: 72nd Annual International Meeting, SEG, Expanded Abstracts, 649–652.

Haber, E., U. Ascher, and D. Oldenburg, 2000, On optimization techniques for solving nonlinear inverse problems: Inverse Problems, **16**, no. 5, 1263–1280, doi:10.1088/0266-5611/16/5/309.

Holtham, E., and D. Oldenburg, 2008, Three-dimensional forward modelling and inversion of Z-TEM data: 78th Annual International Meeting, SEG, Expanded Abstracts, 564-568.

Lo, B., and M. Zang, 2008, Numerical modeling of Z-TEM (airborne AFMAG) responses to guide exploration strategies: 78th Annual International Meeting, SEG, Expanded Abstracts, 1098–1102.

Pellerin, L., J. Johnston, and G. Hohmann, 1996, A numerical evaluation of electromagnetic methods in geothermal exploration: Geophysics, **61**, 121–137, doi:10.1190/1.1443931.

Ward, S., J. O'Donnell, R. Rivera, G. H. Ware, and D. C. Fraser, 1966, AFMAG - applications and limitations: Geophysics, **31**, 576–605, doi:10.1190/1.1439795.

Ward, S. H., 1959, AFMAG—airborne and ground: Geophysics, **24**, 761–787, doi:10.1190/1.1438657.